Thermal Expansion and Contraction in Plastics Piping Systems

PPI TR-21/2001
Thermal Expansion and Contraction
In Plastics Piping Systems

Foreword

This report was developed and published with the technical help and financial support of the members of the PPI (Plastics Pipe Institute, Inc.). The members have shown their interest in quality products by assisting independent standards-making and user organizations in the development of standards, and also by developing reports on an industry-wide basis to help engineers, code officials, specifying groups, and users.

The purpose of this technical report is to provide information on thermal expansion and contraction in plastic piping systems.

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June 2001
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1. INTRODUCTION

Expansion and contraction of piping systems due to temperature changes is not unique to plastics. Changes in temperature tend to cause a change in dimensions of any matter. But, the amount of dimensional change for a given temperature change can vary significantly depending on the material characteristics. Thermoplastic materials exhibit a relatively high tendency to expand and contract when subjected to a temperature change – as much as about ten times that which is exhibited by metals. Accordingly, the design and installation of plastics piping often requires that special attention be paid to this characteristic.

As elaborated later in this report, the restraining of the tendency of a piping system to expand/contract can result in significant stress reactions in pipe and fittings, or between the piping and its supporting structure. The allowing of a moderate change in length of an installed piping system as a consequence of a temperature change is generally beneficial, regardless of the piping material, in that it tends to reduce and redistribute the stresses that are generated should the tendency for a dimensional change be fully restrained. Thus, allowing controlled expansion/contraction to take place in one part of a piping system is an accepted means to prevent added stresses to rise to levels in other parts of the system that could compromise the performance of, or cause damage to the structural integrity of a piping component, or to the structure which supports the piping. Everyone is familiar with the typical expansion loops that are periodically placed in long pipelines subject to wide temperature changes, such as steam lines and the Alaska oil pipeline. These loops absorb changes in pipe length and thereby mute and redistribute the large stresses that would result if pipe thermal movements had been physically restrained. Similar measures for safely absorbing thermal expansion/contraction reactions need to be taken with thermoplastic piping systems.

In installations, such as in buried piping systems, where loops or other means for absorbing thermal expansion/contraction forces are either not an option nor practical, other measures can be used to safely isolate pipe joints, fittings and other piping components from excessive forces.

This report discusses thermal expansion/contraction effects in thermoplastics piping and presents general recommendations for ensuring that these effects are adequately compensated both during design and installation. As demonstrated by laboratory tests and extensive field experience, the potential expansion/contraction problems in plastics piping systems are not as great as their larger tendency to expand and contract due to changes in temperature might suggest. Potential problems can readily be mitigated by the exercise of relatively simple measures that may be taken during piping system design and installation.

The potential consequences of larger expansion/contraction coefficients are greatly offset by plastics’ lower stiffness as well as by their capacity to gradually reduce stress reactions through the phenomenon of stress-relaxation. These qualities of plastics piping result in considerably lower ultimate reaction loads than those that result in metal piping for an equal temperature change. Notwithstanding these beneficial features, design and installation should always be conducted with the objective in mind of the avoiding, or adequately compensating, potentially adverse effects of expansion/contraction. Some situations that require special attention include the following: when unconstrained pipe contraction can
risk the pull-out of a pipe from a fitting; when pipe expansion/contraction can create excessive thrust or bending moments on fittings, particularly on fabricated fittings, flanges and other piping components that are more vulnerable than pipe to such forces; when repetitive expansion/contraction can induce material fatigue at some stress-riser in the piping, typically at some point in a change-of-direction fitting; and when expansion of above ground piping can result in excessive sag.

The information presented herein is intended to serve as a general guide for the identifying and mitigating of potential expansion/contraction problems in the more common applications of plastic piping systems. It should be recognized that a piping system is subject to stresses generated by other reactions than just to temperature variations. For example, in pressure applications the stresses resulting from internal pressure can be a major contributor. Other significant contributions to total stress can come from the weight of the pipe and its contents, from inertial reactions to sudden changes in velocity, and from externally applied loads. For more complex piping systems, especially in which relatively severe service and installation conditions could be encountered, appropriate engineering references should be consulted for more comprehensive guidance for piping system stress/strain analysis and for methods of reducing operating stresses/strains to acceptable values.

2. LINEAR COEFFICIENTS OF THERMAL EXPANSION

The change in length with temperature for a solid material relative to its original length is generally expressed by the parameter linear coefficient of thermal expansion. This parameter has units of reciprocal temperature \([°C]^{-1}\) or \([°F]^{-1}\). Linear thermal expansion/contraction coefficients for commonly used thermoplastics piping materials are presented in Table A-1. The values for plastic materials have been established in accordance with method ASTM D696. Of course, heating or cooling affects all dimensions in a body, with a resultant change in volume. For materials, such as thermoplastics, for which for most practical purposes thermal expansion may be assumed to be isotropic the volumetric coefficient of thermal expansion is approximately three times the linear coefficient.

The values in Table A-1 are for the convenient temperature range of approximately -30°C (-22°F) to +30°C (86°F). The magnitude of the coefficient tends to somewhat increase with increasing temperature. The samples on which the coefficients are established have to be prepared so as to minimize any possible anisotropy and residual stress. In reality, some anisotropy in thermoplastics pipe and fittings always present because of the way in which they are produced – for example, a pipe may have a slightly different coefficient of expansion in the extrusion (i.e., axial) direction as compared to that in its diametrical direction. However, for purposes of design this relatively small degree of anisotropy can generally be neglected. Method of processing could also result in a residual manufacturing stress that can produce some irreversible dimensional change upon the first few temperature change cycles.

It should also be considered that the values in Table A-1 are for fully unrestrained specimens, which is not always the case in actual practice as some restraint against
completely free pipe movement is frequently present. Furthermore, the coefficients given in this table represent typical values for a family of materials. The actual value for a particular commercial plastic may be slightly greater or smaller depending on the material's specific formulation; that is, the kind and amount of other ingredients (i.e., additives) that may have been used in addition to the base polymer. Notwithstanding these limitations, the values in Table A-1 are considered sufficiently reasonable approximations for adequately estimating the effects of expansion/contraction for most cases of plastic piping design and installation.

3. THERMAL EXPANSION/CONTRACTION REACTIONS IN PLASTICS PIPING SYSTEMS

The total stress and total strain that occurs at any point in a piping system, as well as the principal direction in which it acts (i.e., circumferential, axial or radial) depends on the combined effect of several possible reactions. Unless muted by proper design and installation, thermal stress can be a major contributor to the net stress acting in the axial direction. As explained in the introduction, this report only addresses effects of thermal expansion/contraction. The reader should keep in mind that the pipe and each of the other components in a piping system will also be subjected to other stress/strain reactions than those that just arise from thermal expansion/contraction.

Changes in Length – When a pipe is anchored at one end but can otherwise freely move in the axial direction, as is illustrated by Figure 1, an increase in temperature causes the pipe to increase in overall length. A decrease in temperature would cause an opposite change. The following expression predicts the net expansion/contraction in the length of a fully unrestrained pipe that occurs in consequence of a given change in temperature:

\[ \Delta L = \alpha \cdot L \cdot \Delta T \]  

(Equation 1)

Where:

- \( \Delta L \) = change in pipe length, in
- \( L \) = initial pipe length, in
- \( \Delta T \) = change in pipe temperature, °F
- \( \alpha \) = coefficient of linear expansion/contraction, in/in °F

As indicated by the last column in Table A-1, the relatively large coefficient of thermal expansion/contraction of plastics pipe results in a change in length for the same temperature change that is almost ten times greater than that for metal pipe.

The actual change in pipe length occasioned by a temperature change will more often than not somewhat less than calculated by the above equation because installed pipe generally experiences some restraint against freely moving in the axial direction. The restraint may be relatively minor, such as the small frictional drag of guides and supports in the case of above ground installations; or it may be major, as when a pipe is continuously constrained by surrounding soil.
A pipe anchored at one end will expand in length when subjected to a higher temperature.

A pipe restrained from shortening or lengthening will react to temperature changes by generating compressive or tensile forces that act in the axial direction.

In this layout, thermal expansion of the longer vertical run results in the development of high bending stresses in the shorter horizontal run.

Two techniques for isolating a short branch from excessive bending are shown by the use of a loop, which safely absorbs the vertical expansion, and by the placement of a guide, or support, in the short horizontal run to protect it against bending.
Thermally Induced Loads When Pipe is Fully Restrained – In the case of a pipe length which is restrained against lengthening or shortening, a change in pipe temperature results in the development of an axial compressive or tensile stress. This stress is resisted by a compressive or tensile force that acts on the pipe ends, or at any point in the pipe at which it may be anchored. Should this reaction stress and resultant force be sufficiently large it could lead to performance problems in, or damage to a piping component, or to the supporting structure.

The primary concern with plastics pipe in above ground installations is the same as for metal pipe; it is the possibility that an excessively large thrust may be transmitted to a piping components or, to the supporting structure. However, as already pointed out the thermal reaction thrust in plastics piping is generally of much lower magnitude than that which is generated in a metal piping systems for the same temperature differential. Also, because plastic pipe is more flexible than metal pipe – due to the significantly lower elastic modulus of plastic materials – when placed in compression it can more easily deform laterally and when allowed to do so it is somewhat less likely to fully transmit compressive loads onto external restraints than does metal pipe.

The magnitude of the tensile or compressive axial stress that can be generated in a pipe that is constrained both against axial and lateral deformation (See Figure 2) may be computed using the same equation, as follows, as is used with metal pipe:

\[ S = E \alpha \Delta T \]  \hspace{1cm} (Equation 2)

Where:
- \( S \) = tensile or compressive stress in axial direction, psi
- \( E \) = modulus of elasticity (effective modulus in the case of plastics), psi
- \( \alpha \) = coefficient of expansion/contraction, in / in °F
- \( \Delta T \) = temperature differential, °F

There is however, one essential difference in the manner this equation is used with plastics pipe: the effective modulus is used with plastics, whereas with metals the elastic modulus is employed. The term elastic denotes a proportionality between stress and strain that remains constant independent of stress and duration of stress application. While for metals this assumption of elastic stress/strain response is sufficiently accurate for most design purposes it is not applicable to plastics. Plastics are viscoelastic materials – meaning that they behave as a composite of an elastic solid and a highly viscous liquid. Viscoelastic materials exhibit a stress/strain relationship that is dependent on the magnitude of the applied stress and that is also profoundly affected by duration of loading and by temperature. As a result, plastics do not have a true modulus of elasticity. Upon the application of a stress a plastics material undergoes an initial resultant deformation which upon increased duration of loading grows – but at a gradually decreasing rate – into an ever larger deformation. This continuing, but steadily slowing down deformation under continuous loading is called creep.

When subjected to constant deformation there is an analogous response called stress-relaxation. The stress required to maintain a certain initial deformation tends to diminish, or relax, with time. As in the case of creep, stress-relaxation is fairly rapid at first, but as time increases the rate of stress-relaxation decreases and asymptotically approaches a
final value. Both rate of creep and rate of stress-relaxation in thermoplastics are greatly affected by temperature.

Even though the stress/strain relationship of viscoelastic material is much more complex than that of elastic materials the same equations which have been developed for the latter can be used with the former by means of the concept of effective modulus. Effective modulus (sometimes also referred to as the creep-modulus) is the approximate ratio of stress to strain that results under a particular combination of conditions (e.g., magnitude of load, duration of loading, temperature and environment). Consequently, whenever a value of effective modulus is reported the applicable conditions (e.g., 50-years under continuous loading at 73°F) are also stated. Often times the effective modulus is expressed as a percentage of the material’s short-term modulus which has been determined by means of a standard short duration tensile strength test that takes only minutes to complete. Approximately values of the ambient temperature short-term moduli of various piping material are presented in Table A-2. Also included in this table are multipliers by which these values may be reduced to approximate values of effective modulus for certain time periods of continuous loading. These multipliers may also be used as an index of the rate at which stress-relaxation occurs under the condition of constant deformation.

Approximate temperature multipliers by which the values in Table A-2 may be adjusted to other temperatures are shown in Table A-3.

An often used convention for the estimating of the initial (i.e., maximum) axial stress that can result by constraining pipe expansion/contraction is to conduct the calculation under two generally very conservative assumptions: the temperature change occurs very quickly, meaning that the short-term modulus should be used; and the material stiffness for the lower temperature is the primary determinant of the magnitude of the generated stress, meaning that the short-term modulus for the lowest temperature should be used. Because the effective modulus in fact decreases with increasing temperature and because the change in temperature seldom occurs very quickly, both these assumptions lead to an estimate of that is somewhat larger than actual.

For an example of the application of Equation 2 let us assume that a PE 3408 pipe, which because it has been laying above ground in direct exposure to the sun is at a temperature of 120°F, is being lowered into the shaded bottom of a trench that has been excavated in -term modulus for this pipe material of 130,000 psi for 50°F (interpolated from the values given in Table A-3) and using the expansion contraction coefficient of 9.0 x 10⁻⁵ in/in x °F (from Table A-1) the estimated maximum initial axial tensile stress than can occur in this pipe if it were restrained against contraction immediately after placement in the cooler trench (by say, being connected to an anchored fitting) is as follows:

\[ S = E \cdot \alpha \cdot \Delta T \]

\[ S = 130,000 \text{ psi} \cdot 9.0 \times 10^{-5} \text{ (°F)}^{-1} \cdot (120 - 50) \text{ °F} \]

\[ S = 819 \text{ psi} \]

As pointed out above, the actual initially developed stress will be somewhat lower. However, even this overstated value is well within both the short-term strength and the long-term strength capabilities of PE 3408 materials which, respectively, are about 3,200 and
1,600 psi for water, for 73.4 °F (see Table A-4). Since strength increases with decreasing temperature, both these strength values are somewhat greater at the ground temperature of 50°F. In order to generate an initial stress that is about the same as the long-term strength of this material the temperature differential would have to be twice as large as that assumed for this sample calculation, an unlikely condition. And even a much higher temperature differential would be required to challenge this material’s short-term strength. Similar evaluations for other thermoplastic pipes have shown that the thermal stress that may be induced in common applications of plastics piping is seldom large enough to be a challenge to the pipe’s strength properties. However, in combination with stresses that can result from other causes – such as internal pressure – a high thermal stress could produce a combined axial stress that could be sufficiently large to impose excessive forces on the other components of a piping system, such as joints, fittings and also, possibly, on the structures that support the piping.

It should be noted from inspection of Equation 2, that the axial stress that is induced in a pipe that is restrained from freely expanding or contracting is independent of pipe length. However, as indicated by Equation 1, the length of pipe that is unrestrained against axial deformation is a determinant of the extent of the deformation at the end of a pipe length.

The thermal thrust in a pipe that is constrained against axial movement may be calculated from the axial stress by means of the following equation:

\[
F = S \cdot A
\]  

*(Equation 3)*

Where:  
\(F\) = thrust (tensile or compressive), lbs  
\(S\) = tensile or compressive stress in accordance with Equation 1, psi  
\(A\) = cross sectional area of pipe perpendicular to stress direction, in\(^2\)

Let us assume that in the previous example the PE 3408 pipe is 4-in nominal diameter (actual outside average diameter is 4.500 in), of SDR 21 construction (minimum wall thickness is 0.214 in), and that one end of the pipe has been anchored by some means and the other end has been connected to a rigidly held compression fitting (e.g., to a metal tap that has been placed on a well anchored larger diameter steel main) very soon after the pipe was lowered into the much cooler trench. As the pipe cools from 120 to 50°F its tendency to contract is constrained which generates an axial tensile load reaction that tends to want to pull the pipe out of the compression fitting. In accordance with the above equation the initial pullout force, or tensile thrust, can be as large as:

\[
F = \text{Maximum axial tensile force} \cdot \text{Pipe annular cross sectional area}
\]

\[
F = 819 \text{ psi} \cdot \pi \cdot (4.500 - 0.214) \cdot 0.214 \text{ in}^2 = 2,360 \text{ lbs}
\]

If the compression fitting is unable to resist this force then the PE pipe will begin to pullout. And if the pipe is of relatively long length, this tensile pulling force could cause sufficient withdrawal of the pipe from the fitting to result in its complete pullout. To avoid this consequence the final connection should not be made until the pipe has cooled to
approximately ground temperature. Another preventative measure is to use a pull-out resistant mechanical fitting. These and other means for mitigating problems resulting from thermal effects are discussed in subsequent sections of this report.

**Axial Pull-Out Forces Acting on Pipe Joints, Fittings and Other Components**

While rare is the occasion where the combined thrust due to thermal effects and other causes can be sufficient to challenge the axial strength of a plastic pipe this is not always the case for pipe joints, fittings and other components. Some pipe joints have relatively low resistance to pipe pullout; and some piping components, such as plastic pipe flanges, may have relatively low strength in the axial direction. If the total anticipated axial thrust cannot be adequately resisted by a pipe joint or by a piping component, then it must either be diminished to some safe value by building-in additional flexibility into the piping layout so as to transform any potentially excessive thrust into a more tolerable bending force; or by isolating all vulnerable piping components from excessive axial thrust by means of properly placed anchors or other devises.

Joints of plastic to plastic which are made by the heat fusion process are characterized by high axial strength – generally, they are as strong as the pipe. Heat-fusion and solvent cemented connections using the bell-spigot (or, socket-spigot) design are similarly strong in pipe sizes up to 4-inch. In the larger diameters, where socket depth tends to decrease with increasing pipe diameter, the user should contact the pipe or fitting supplier to determine axial thrust limitations.

The thrust tolerance, or joint strength of mechanical fittings can vary widely depending on fitting design. Certain mechanical fittings have been designed to offer axial strength that is equal to or greater than that offered by the plastic pipe for which they are intended. Other connectors, like many of the rubber-gasket bell and spigot joints, are designed primarily for pressure sealing and less so for holding. Consequently, such fittings are almost always used in under-ground installations in which the piping can readily be restrained against excessive axial and lateral movement. And, there are many designs in between.

Mechanical fittings intended to be used in aboveground systems, such as for flexible plastics piping, are generally designed to withstand the total anticipated axial thrusts that can result from internal pressure and other causes. However, because of the wide array of designs of mechanical fittings it is the responsibility of the specifier and installer to make sure that the axial, or the pullout strength of a mechanical fitting is adequate for the intended application and service conditions.

To preclude the risk of a pullout of a plastic gas pipe from a fitting, the U.S. Department of Transportation Pipeline Safety Regulations that govern the design and installation of underground fuel gas piping require that “the pipeline must be designed and installed so that each joint will sustain the longitudinal pull-out or thrust forces caused by contraction or expansion of the piping or by anticipated external or internal loading (Paragraph 192.273, Chapter. I, Volume 49, Code of Federal Regulations)”. In consideration of this requirement, ASTM D2513, *Standard Specification for Thermoplastic Pipe, Tubing and Fittings*, classifies mechanical fittings into three categories, as follows, in accordance with their capacity to resist pullout:
**Category 1** – A mechanical joint design that provides a seal plus a resistance to a force on the pipe end equal to or greater than that which will cause a permanent deformation of the pipe.

**Category 2** – A mechanical joint design that provides a seal only. A mechanical joint designed for this category excludes any provisions in the design and installation of the joint to resist any axial pullout forces; therefore tensile test are not required.

**Category 3** – A mechanical joint design that provides a seal plus a pipe restraint rating equivalent to the anticipated thermal stress occurring in a pipeline. This category has a manufacturer’s rated pipe end restraint less than the value required to yield the pipe as outlined in Category 1.

Manufacturers of compression fittings intended for gas distribution applications evaluate and rate the pullout resistance of their products in accordance with the above categories. When using Category 3 fittings, the aforementioned Federal Code holds the user responsible for the computing of the “anticipated thermal stress” and the resultant force that can develop in the pipeline that is being installed, and also for ensuring that this resultant force can be safely tolerated by the fitting which is to be used. In an Annex on Thermal Stress, ASTM D2513 details the procedure and assumptions for the computing of the anticipated thermal contraction (i.e., pullout) force and of the magnitude of the pipe contraction which could occur under this force. The equations used for this calculation are the same as previously given in this report, namely Equations 1, 2 and 3. ASTM D2513 calls for the following assumptions when making the calculations:

1. *There are no forces restraining the pipe against thermal contraction.* While this assumption is reasonable for insert applications, it is conservative for direct buried applications in which the surrounding soil does provide restraint.

2. *The temperature differential driving contraction is equal to the maximum temperature minus the minimum temperature that are expected to be seen by the pipe in the subject installation.* Implicit to this is the assumption that a pipe contraction force will always be present when the pipe is at less than its maximum anticipated service temperature. The fact is that in nearly all cases active resistance against contraction will not begin until the pipe is at a temperature that is lower than that at which the last pipe connection was made.

3. *The instantaneous modulus of elasticity is used.* Using this short-term value implicitly assumes that that the change in temperature from maximum to minimum occurs almost instantaneously. The fact is that in buried installations temperature changes occur relatively slowly, and often times they follow the slowly changing seasonal changes in ground temperature.

**Bending Moments Induced by Thermal Expansion/Contraction** – Above ground piping systems are often laid out so as to include a number of changes in direction. In such systems thermal pipe expansion/contraction induces bending stresses that need to be
limited so as not to risk a structural damage to a piping component. A layout that invites the development of high-localized bending stresses is illustrated in Figure 3. Piping runs should be designed in such a way as to preclude the possibility of developing excessive bending stresses, particularly in fittings which may have significantly lower tolerance to bending forces than does pipe.

There is a very beneficial aspect of the inclusion of changes in direction in a piping system. As shown by Figure 4, directional changes create a spring-like effect that can be used to safely absorb the thermal expansion/contraction that occurs when a piping run is subjected to changes in temperature. In systems in which this spring-like effect is not a natural component its benefits can be incorporated through thoughtful system design. The sections which follow present guidelines and suggestions for loops, offsets and other devices for safely building-in adequate flexibility into a plastics piping system.

4. COMMONLY EMPLOYED MEANS FOR MITIGATING EFFECTS OF THERMAL EXPANSION/CONTRACTION

A – IN UNDERGROUND APPLICATIONS

Direct Burial Installations – For most direct burial applications the effects of thermal expansion/contraction can be readily mitigated through proper installation and by using, where required, joints and fittings that have adequate resistance to axial thrust. When these measures are taken, thermal expansion/contraction reactions are seldom a significant factor in pipeline design and operation. A buried, or a concrete encased pipe is effectively restrained both against lateral and axial movement by the surrounding embedment material. The magnitude of the frictional restraining force is dependent on the nature of the soil and on installation and operating conditions. For example, the extent of compaction near the pipe can affect the quality of contact between the pipe and the surrounding soil. Sometimes the quality of contact between pipe and surrounding soil could be lessened by wide ranging operating pressure and temperature fluctuations, both of which could cause the pipe’s outside diameter to cyclically expand and contract.

The “anchoring”, or restraining effect of surrounding soil on pipe movement can be significantly augmented by external pipe geometry. Tees, lateral connections, and changes in direction all help to anchor a pipe in the surrounding soil. The larger outside diameter of the bell section of bell and spigot plastic pipe acts as an effective anchor. With bell and spigot piping thermal expansion/contraction is seldom a factor in system design and operation.

Because the friction between the pipe and surrounding material is generally sufficient to arrest axial pipe movement, a buried pipe that is subject to typical fluctuations in the temperature of the fluid it conveys or of the soil that surrounds it is only subject to modest axial thermal stresses that are well within the strength capabilities of the pipe. However, the possibility should always be considered that a thermal stress that is easily tolerated by a pipe may be transmitted to a piping system joint, fitting or other components that may have lesser strength, or that may be susceptible to pipe pull-out. Particular care should be taken with piping components that may have modest axial strength, such as certain flange
connections and fabricated fittings, to make sure they are adequately isolated from excessive axial thrust. The designer and installer should be aware of the axial, bending or other strength limitation of all components in a piping system. In addition, some fittings may have low tolerance to bending stresses.

To minimize the development of contraction axial stresses in newly installed pipe, the pipe should be allowed to cool to approximately ground temperature before backfilling and before making final connections to rigidly held fittings. When the potential for high thrust is indicated, or where there might exist strength or pullout limitations in fittings, other means such as illustrated by Figure 5 should be used to isolate a vulnerable joint or fitting from excessive stressing.

The magnitude of the soil restraint, which acts on plastic pipe with an externally smooth wall, may be estimated from the following equation:

\[ f = \mu \cdot N \]  

*(Equation 4)*

Where:

- \( f \) = axial frictional resistance, lbs/in of pipe length
- \( \mu \) = approximate coefficient of friction between soil and pipe, and between concrete and pipe. A value of 0.1 is generally accepted as a conservative representation for the case where smooth surface plastic pipe makes full contact with the embedment material
- \( N \) = normal soil pressure acting on 1-in of width of pipe, psi/in

\[ N = \pi \cdot D_o \cdot \text{Soil pressure} \]

Where \( D_o \) is the pipe’s outside diameter, in

**Insertion, or “Slip-Line” Applications** – In applications where defective buried pipes have been renewed by the insertion of new plastic pipes (i.e., by the use of so-called “insert renewal”, or “slip-lining” procedures) there often is little, if any, soil or other material that surrounds the inserted plastics pipe. A tendency for the plastic pipe to contract, such as may occur when the pipe is subjected to a significantly lower operating temperature than that at which it was installed, could transmit a substantial tensile thrust onto rigidly held fittings, or other appurtenances and structures to which the pipe is connected. When restraint by embedment material is absent, or is minimally present, special attention needs to be given to ensure that pipe thermal pipe reaction will not generate excessive thrust or pullout forces. As pointed out in the discussion on direct buried pipe, the final connection to a rigidly held fitting should be made not until after a newly installed plastic pipe has come to about the same temperature as that of the material that surrounds it; and the adequacy of the pull-out resistance of the joints that connect the pipe to rigidly held structures should be given due consideration.

However, in slip-line installations there is another significant factor to consider. Long lengths of plastic pipe are often pulled into existing pipes by means of winches or other mechanical devices. The pulling force may slightly elongate the pipe during the process of insertion. Upon removal of this pulling force there may be left in the pipe a gradually decreasing residual axial stress which is acting to return the pipe to its original unstretched
length. If this return were to be constrained by promptly connecting the liner to a rigidly held connection there would gradually build-up in the pipe a tensile stress that would impress a pulling force on the connections at the ends of the liner. This gives further reason for allowing a newly inserted pipeline to reach equilibrium before making the final tie-ins. A typical recommendation is to allow plastic liners that have pulled, or pushed into place to relax for at least 24 hours.

Figure 5

Flange Bending Protection

Fabricated Fitting Bending Protection

[Other sketches to be supplied by PPI members. Possible sketches of interest: the heat fusing of saddles on a PE pipe and the anchoring of this section by means of poured concrete; use of thrust blocks.]

Examples of methods that are used to isolate joints and fittings from excessive bending and pullout forces.

B – IN ABOVE GROUND APPLICATIONS

Introduction – Above ground piping generally undergoes more temperature changes than underground piping because of atmospheric temperature changes and the greater temperature variations of the fluids being conveyed. Two examples are hot/cold water service pipes that carry hot water intermittently with cold water in between, and a chemical transfer line through which a hot fluid may flow intermittently. However, in many such applications the effect of such wide swings in temperature are often satisfactorily offset because in most such installations the piping consists of relatively short runs – generally, under 20 feet in length – with changes in direction which endow the system with a natural flexibility. In such cases, no special precautions need be taken in the piping runs to offset expansion/contraction reactions. However, where this flexibility is employed the pipe movement should not be constrained by external obstructions, such as by placing an elbow or other bend too closely to a wall or other object. Excepting at points at which the pipe is purposely anchored, it must be able to move in guides and hangers. And care should
always be taken to ensure that the pipe layout, including the placement of anchors\(^1\) and guides\(^2\) adequately isolates a joint, fitting or connection from undue tensile thrust or bending moment. As previously discussed, certain compression joints may have relatively low pullout resistance. And most fittings have lower bending strength than does the pipe for which they are intended. Some piping components such as fabricated fittings, saddles and flanges may have very significantly lower axial and bending strength. In the case of such components particular care should be taken to adequately isolate them against excessive pullout and bending stresses. Some examples of incorrect piping layouts and suggested corrections are shown on Figure 6.

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\(^1\) Anchor – Anchors are used in piping systems as fixed points from which to direct expansion/contraction and other movements in a defined direction.

\(^2\) Guides – Guides are used to allow axial motion while preventing transverse movement. Both anchors and guides may be used in the control of expansion/contraction of pipelines.
Figure 6

Incorrect                      Correct

\[\text{Diagram images of incorrect and correct examples.}\]
V = Valve (Anchor Point)
B = Bracket (Guide)

Some examples of pipe layouts with insufficient flexibility and suggested corrections.

In situations where the piping makes relatively long runs or where it may be exposed to large temperature swings the higher resultant expansion/contraction will usually have to be absorbed by additional means. When the piping is dark colored – as in the case of black PE piping which has been made sun light resistant by the addition of finely divided carbon black – direct exposure to the sun may result in pipe surface temperatures as high as 140°F which in some places may drop to subzero values at nighttime. In such cases, the larger potential expansion/contraction can be safely absorbed by building-in added flexibility, and some times by the use of expansion joints.

Two kinds of expansion joints have been used with plastics: the packless kind, which consists of a bellows design wherein the bellows are made from a fatigue resistant material; and a slip joint that is kept leak-tight by the use of a packing that can consist of a rubber gasket or other appropriately chemically resistant material. Excepting for some low or non-pressure applications such as for drain, waste and vent piping, expansion joints are not often used. When subjected to a compressive thrust plastic pipe, because of its relatively lower stiffness as compared to metal pipe, tends to deflect laterally rather than generate the reactive thrust necessary to close the expansion joint. In the case of the slip kind of expansion joint, it is difficult to achieve a design that allows return at a low axial compressive thrust while simultaneously providing a seal that remains tight under a broad range of operating temperatures and pressures. In pressure service expansion joints readily expand but do not always fully return, which causes further pipe deflection. Furthermore, because expansion joints do not transmit thrust they need to be used with carefully located thrust absorbing pipe anchors, a requirement that may tend to complicate piping layout and supporting structure design. However, expansion joints are the preferred solution for certain situations such as when it may not be as practical to configure added flexibility into a piping system. Expansion joints are commercially available, principally for PVC and CPVC, in up to about 12-inches pipe diameter.

The more common solution for the absorbing of thermal expansion/contraction in plastic pipelines is to build-in added flexibility. There are two ways this may be done: the first is by the addition of offsets or expansion loops; and the second is by allowing controlled lateral deflection. Offsets and expansion loops are primarily used with pipes that are supported periodically, such as by hangers. Lateral deflection is employed with pipes that have continuous support such as by racks or when installed on grade, which could be either above or below water.

**Offset and Expansion Loop Calculation** – A compressive or tensile axial stress in a straight run of pipe may be relieved by transforming it to a bending stress at an offset. As illustrated by Figure 7, the offset length \( L_o \) acts as a cantilevered beam to the long pipe run \( L_r \). Under thermal expansion, length \( L_r \) increases by \( \Delta L_r \), which forces offset \( L_o \) to bend, and thereby absorb the expansion stresses. The length of the offset, \( L_o \), needs to be sufficient so that the neither the resultant bending stresses nor the resultant bending
moments exceed the working strength limits of the offset, including that of all included fittings.

Figure 7

A pipe offset relieves an axial thrust by transforming it into a moderate bending stress.

Figure 8

The minimum calculated length of offset may be distributed as segments of a loop or other means for imparting flexibility. Recommended minimum lengths for each segment are illustrated above. L is the total offset length as determined by means of Equation 5.

The minimum length of offset, \( L_0 \), that will safely absorb the thermal load in a long straight length of pipe, \( L_r \), may be calculated from the equation that follows. (see Appendix for derivation):
\[ L_o = \left[ \frac{3}{2} \cdot \frac{E}{S} \right]^{1/2} \cdot \left[ D \cdot \Delta L_r \right]^{1/2} \]  

(Equation 5)

Where:

- \( L_o \) = Minimum length of offset leg, in
- \( E \) = Effective modulus for range of operating temperature, psi
- \( S \) = Maximum allowable bending stress for range of anticipated operating conditions, psi
- \( D \) = Outside pipe diameter, in
- \( \Delta L_r \) = Anticipated maximum change in length in pipe length \( L_r \), in (calculated by means of Equation 1)

After the minimum length \( L \) is determined, these same equations may be used to determine whether the guide in the main run is sufficiently away from the 90° directional change to ensure that the bending stresses in this section of the main run are also below the limiting value. To conduct this calculation, one will first have to calculate \( \Delta L \), the anticipated increase in length in the offset.

It should be noted that in Figure 7, the bending in the offset begins at a guide and not at an anchor point. As already mentioned, some fittings have relatively low tolerance to bending, particularly when the bending is repetitive which can lower strength through fatigue. Accordingly, connections of offsets to rigidly held fittings should be isolated from bending stresses by means of guides, clamps or other devices.

Since the ratio \( S/E \) depicts the resultant strain, Equation 5 may also be written as follows:

\[ L_o = \left[ \frac{3}{2} \cdot \frac{1}{\varepsilon} \right]^{1/2} \cdot \left[ D \cdot \Delta L_r \right]^{1/2} \]  

(Equation 6)

Where:

- \( \varepsilon \) = \( S/E \) = Allowable fiber strain, inch/inch

It is up to the designer to select the appropriate values of material constants when conducting the calculation in accordance with the above equations. However, an often-used practice is to assume that the temperature change occurs very quickly but that the induced bending stress will be long lasting. Accordingly, the short-term modulus and the long-term design stress are often used for \( E \) and \( S \), a combination that results in a very conservative estimate of minimum required length of offset. Another common practice is to use the modulus and strength values for the lowest temperature. This usually results in the longest offset. As the temperature increases, both the effective modulus and the maximum allowable strength of plastic piping materials tends to decrease at about the same rate but more often than not the modulus drops off a little faster, which leads to a somewhat greater tolerance to bending at the higher temperatures. The exact rate of decrease of each of these properties with increasing temperature can vary among different materials, and even among materials within the same plastics family. However, when the conservative assumptions of short-term modulus and long-term strength are used to calculate minimum offset length, the ignoring of the differences in rate of change of \( E \) and \( S \) within a material’s allowable operating temperature range has no significant consequence on the result.
The ratio of the long-term strength to the short-term modulus of most thermoplastic piping materials lies around the value of 0.005 inch/inch. Some manufacturers of plastics piping have adopted this value as the limiting strain for calculating minimum offset length. Based on this value of $\varepsilon$, Equation 6 can be simplified to the following:

$$L_o = 17.3 \left[ D \cdot \Delta L_r \right]^{1/2}, \text{ or }$$

$$l_o = 1.44 \left[ D \cdot \Delta L_r \right]^{1/2} \quad \text{(Equation 7)}$$

Where: $l_o =$ offset length expressed in feet

A graphical solution to Equation 7 for various assumed values of limiting strain is included in the Appendix.

Relief of expansion/contraction reactions may be accomplished by other configurations than offsets, such as loops and changes in direction, so long as: 1) the developed length in these configurations is as least as long as that required for a simple offset; and 2) the length of each segment of the configuration is sufficiently long to preclude the development of excessive bending stresses. Guidelines for the achieving of these objectives have been developed and are illustrated in Figure 8.

**On-Grade Installations** – Thermal expansion in longer runs of piping that are supported continuously, either when placed on a ground surface or in pipe rack, can be absorbed by allowing the pipe to deflect laterally. In such cases there must be sufficient supporting room on either side of the pipe to accommodate lateral deflection. The pipe is confined in its right-of-way by anchoring it periodically. In pipe racks center anchors may be used but these must be able to pivot with pipe deflection. In a pipe run along a surface the pipe may be anchored by placing it between paired posts. When deflection is to one side only, posts need to be placed on only one side.

To direct the pipe to only deflect to one side it can be laid with an initial deflection that undulates from one side to the other so that additional deflection will always continue in the same direction. An initial deflection should be provided so that when at its lowest anticipated temperature the pipe will not contract to straight and become subject to a high axial tensile thrust. To achieve the initial required lateral deflection, at the time of installation should be determined the anticipated change from installation temperature to minimum temperature. Using this value and the distance between lateral support points the lateral deflection at the installation temperature may be determined and the pipe is then installed with this lateral deflection plus any added lateral deflection that may be specified by the designer.

The surface over which the pipe deflects should be free of large rocks, projecting stones, debris, clods or other material that may damage the pipe and thereby compromise the pipe material’s rated strain capacity.

Lateral deflection of an end-constrained pipe may be determined from the following relationship:

$$y = L \left[ \frac{1}{2} \alpha \cdot \Delta T \right]^{1/2} \quad \text{(Equation 8)}$$
Where:

- \( y \) = lateral deflection, in
- \( L \) = distance between lateral supports, in
- \( \alpha \) = thermal linear expansion coefficient, in/in • °F
- \( \Delta T \) = change in temperature, °F

There needs to be sufficient free space on either side of the pipe to allow this deflection to freely take place.

The closer the lateral supports are placed to each other the greater will be the pipe curvature as a result of lateral bending. Spacing of lateral supports should not be so frequent as to result in excessive bending stresses or strains in the pipe, including the pipe joints. The following equation may be used to calculate the minimum distance between restraining points:

\[
L_m = 0.098 \cdot D \cdot \frac{E}{S \cdot \Delta T^{1/2}}
\]

(Equation 9)

Where:

- \( L_m \) = minimum spacing, in
- \( D \) = pipe outside diameter, in

(the other symbols have been previously defined)

Since the ratio \( S/E \) is equal to \( \varepsilon \), the resultant strain, the above equation may be written as:

\[
L_m = 0.098 \cdot D \cdot \frac{1}{\varepsilon \cdot \Delta T^{1/2}}
\]

(Equation 10)

The choice of the value for \( \varepsilon \) should give consideration to the strain capacity of the pipe plus any included fittings and joints. For example, in the case of PE piping some manufacturers recommend that when a pipe bend includes a fitting the pipe bending radius, in inches, should not be tighter than 100 times the pipe’s outside diameter, also in inches. This practice is equivalent to the limiting of fiber strain to a maximum of 0.01, or 1%. Oftentimes, a larger limiting strain may be recommended by the manufacturer; in some cases as high as 5%. The manufacturer should be consulted for recommended values.

**Installations of Flexible Tubing** – Because of its natural flexibility, flexible tubing is able to readily absorb expansion/contraction reactions. The installation should anticipate and accommodate this reaction by laying out the tubing run in such fashion as to avoid undue interference to free movement by walls, beams or other objects. A consequence of a plastic tubing making heavy contact with some building structure is that upon movement the tubing can produce a ticking or rubbing sounds that may be annoying to the occupants.

There is no structural problem associated with deflection of tubing. A deflection caused by a change in length is mostly an aesthetic problem. However, in most installations the tubing is concealed inside walls, ceilings, attics and other such spaces. When not concealed, tubing may be supported with greater frequency to create a more acceptable aesthetic appearance. As in the case of all plastic pipe, tubing that is constrained from expanding/contracting by its being encased in concrete, soil or other material will readily absorb any resultant stresses.
**Drain, Waste & Vent (DWV) Installations** – Expansion/contraction does not present any problems in DWV installations in structures up to 2 stories. It can be a cause of design and installation concern in taller structures principally because long stacks are involved. Expansion of a tall stack could lift the laterals that are connected to it in the upper stories to a sufficient extent as to impair their capacity to freely drain or produce excessive stress/strain in stack fittings. Three methods, as follows, each of which has been previously described, may be used to control expansion in DWV stacks:

*By the use of offsets* – The minimum length of the offset that should be provided may be calculated using Equation 5. This calculation is generally made under the same assumptions as used for pressure pipe. For example, by assuming a limiting strain of 0.005 to 0.010 inch/inch.

*By the use of expansion joints* – Where allowed by the applicable code, expansion joints may be used. Expansion joints for ABS and PVC stacks are available in sizes 6” and smaller. These should be installed approximately every 30 feet. Unless it is installed at one of the extreme anticipated service temperatures, the expansion joint is generally installed in the neutral position so that in normal service it can move in either direction. On vertical stacks, the pipe above the joint should be anchored near the expansion joint to prevent it from telescoping into the joint because of the weight of the stack above this point.

*By the use of restraints* – Engineering studies have shown that by applying restraints to vertical movement every 30 feet the tensile and compressive forces that are developed in a stack through this restraint are readily absorbed by the piping without any damage.
## APPENDIX

### TABLE A-1

LINEAR COEFFICIENTS OF LINEAR THERMAL EXPANSION/CONTRACTION FOR COMMONLY USED PLASTICS AND METAL PIPING MATERIALS

<table>
<thead>
<tr>
<th>Piping Material (Plastic Pipe Pressure Grades Identified by Parenthesis) (1)</th>
<th>Coefficient of Linear Thermal Expansion, ASTM D696 (in/in x °F)</th>
<th>Resultant Pipe Expansion (inches/100 feet x 10°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>6.7 x 10^{-6}</td>
<td>0.08</td>
</tr>
<tr>
<td>Cast Iron</td>
<td>5.9 x 10^{-6}</td>
<td>0.07</td>
</tr>
<tr>
<td>Copper</td>
<td>9.3 x 10^{-6}</td>
<td>0.11</td>
</tr>
<tr>
<td>PE, Medium Density (PE 2406)</td>
<td>9.0 x 10^{-5}</td>
<td>1.10</td>
</tr>
<tr>
<td>PE, High Density (PE 3408)</td>
<td>9.0 x 10^{-5}</td>
<td>1.10</td>
</tr>
<tr>
<td>PEX (PEX 0006)</td>
<td>9.5 x 10^{-5}</td>
<td>1.20</td>
</tr>
<tr>
<td>PVC, 12454 &amp; 12364</td>
<td>3 x 10^{-5}</td>
<td>0.36</td>
</tr>
<tr>
<td>PVC, Unmodified (PVC 1120 &amp; 1220)</td>
<td>3.5 x 10^{-5}</td>
<td>0.42</td>
</tr>
<tr>
<td>PVC, Impact Modified (PVC 2110 through 2120)</td>
<td>5.0 x 10^{-5}</td>
<td>0.60</td>
</tr>
<tr>
<td>CPVC (CPVC 4120)</td>
<td>3.4 x 10^{-5}</td>
<td>0.41</td>
</tr>
<tr>
<td>ABS, Type 1, Grade 1</td>
<td>6.0 x 10^{-5}</td>
<td>0.72</td>
</tr>
<tr>
<td>ABS, Type 1, Grade 3</td>
<td>5.2 x 10^{-5}</td>
<td>0.62</td>
</tr>
<tr>
<td>ABS, Type 2, Grade 1</td>
<td>6.0 x 10^{-5}</td>
<td>0.72</td>
</tr>
<tr>
<td>PB, Type 2, Grade 1</td>
<td>7.2 x 10^{-5}</td>
<td>0.86</td>
</tr>
<tr>
<td>PP, Type 1, Grade 2</td>
<td>4.2 x 10^{-5}</td>
<td>0.52</td>
</tr>
<tr>
<td>PP, Type 2, Grade 2</td>
<td>4.3 x 10^{-5}</td>
<td>0.48</td>
</tr>
<tr>
<td>PVDF (PVDF 2025)</td>
<td>7.8 x 10^{-5}</td>
<td>0.93</td>
</tr>
</tbody>
</table>
Abbreviations for plastics are in accordance with ASTM D883 and D1600. They denote the following plastics: PE - polyethylene; PEX - crosslinked polyethylene; PVC - polyvinyl chloride; ABS - acrylonitrile-butadiene-styrene; PB - polybutylene; PP - polypropylene; PVDF - polyvinylidene difluoride

TABLE A-2

APPROXIMATE VALUES OF SHORT-TERM MODULUS FOR AMBIENT TEMPERATURE (73.4°F) FOR COMMONLY USED PRESSURE PIPING MATERIALS AND APPROXIMATE MULTIPLICATION FACTORS BY WHICH TO REDUCE THESE VALUES TO EFFECTIVE MODULUS FOR DIFFERENT DURATIONS OF CONTINUOUS LOADING

<table>
<thead>
<tr>
<th>Pressure Piping Material</th>
<th>Short-term Modules @ 73.4 °F (psi)</th>
<th>1 hr</th>
<th>100 hrs</th>
<th>10,000 hrs</th>
<th>438,000 hrs (50 yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PE 2406</td>
<td>90,000</td>
<td>0.80</td>
<td>0.52</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>PE 3408</td>
<td>110,000</td>
<td>0.80</td>
<td>0.52</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>PVC 1120</td>
<td>420,000</td>
<td>0.84</td>
<td>0.60</td>
<td>0.40</td>
<td>0.34</td>
</tr>
</tbody>
</table>

TABLE A-3

APPROXIMATE MULTIPLICATION FACTORS FOR ESTIMATING EFFECTIVE MODULUS FOR OTHER TEMPERATURES THAN 73.4°F

<table>
<thead>
<tr>
<th>Temperature Compensation Multiplier (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>PE 2406</td>
</tr>
<tr>
<td>PE 3408</td>
</tr>
<tr>
<td>PVC 1120</td>
</tr>
<tr>
<td>CPVC 4120</td>
</tr>
</tbody>
</table>

Values for temperatures within those listed in this table may be estimated by arithmetic interpolation.
TABLE A-4

APPROXIMATE SHORT AND LONG-TERM STRENGTH VALUES FOR COMMON THERMOPLASTICS PIPING MATERIALS FOR 73°F

<table>
<thead>
<tr>
<th>Material</th>
<th>Short-term Strength, psi (ASTM D 638)</th>
<th>Long-term Strength, psi (ASTM D2837)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PE 2406</td>
<td>2,600</td>
<td>1,250</td>
</tr>
<tr>
<td>PE 3408</td>
<td>3,200</td>
<td>1,600</td>
</tr>
<tr>
<td>PEX 0006</td>
<td>2,800</td>
<td>1,250</td>
</tr>
<tr>
<td>PB 2110</td>
<td>4,200</td>
<td>2,000</td>
</tr>
<tr>
<td>PVC 1120</td>
<td>8,000</td>
<td>4,000</td>
</tr>
<tr>
<td>PVC 2116</td>
<td>6,000</td>
<td>3,250</td>
</tr>
<tr>
<td>CPVC 4120</td>
<td>8,000</td>
<td>4,000</td>
</tr>
<tr>
<td>PVDF 2025</td>
<td>7,500</td>
<td>5,000</td>
</tr>
</tbody>
</table>

DEVELOPMENT OF EXPANSION LOOP EQUATION

An offset in a long pipeline can be considered as a cantilevered beam that is anchored at one end and that deflects at the other end in response to the applied force that is generated by the expansion or contraction of pipeline (See Figure 7). The resultant deflection at the free end of the cantilevered beam (the offset) depends on the magnitude of the applied force as well as the structural properties of the beam. The relationship between these various parameters is as follows:\(^3\):

\[ \Delta y = \frac{F L^3}{3EI} \]

Where:

\[ \Delta y = \text{maximum deflection at the end of a cantilevered beam, inch} \]
\[ F = \text{force applied at free end of beam, lbs} \]
\[ L = \text{length of beam, in} \]
\[ E = \text{flexural modulus of elasticity of the beam material, psi} \]
\[ I = \text{moment of inertia for the beam geometry, in}^4/\text{in} \]

The following relationships also apply to cantilevered beams:

\[ S = \frac{Mc}{I} \]

Where:
- \( c = \frac{D}{2}, \text{ in.} \)
- \( M = FL \)
- \( M = \) the bending moment inch-lbs
- \( S = \) maximum fiber stress in bending, psi

Substituting: \( S = \frac{Mc}{I} = \frac{FLD}{I^2} \)

Rearrange: \( I = \frac{FLD}{2S} \)

Substitute for \( I \) in the maximum deflection equation:
\[ \Delta y = \frac{FL^3}{3E(FLD/2S)} \]

Cancel units and rearrange:
\[ \Delta y = \frac{2SL^2}{3ED} \]

Solving for \( L^2 \):
\[ L^2 = \frac{3ED\Delta y}{2S} \]

Solving for \( L \):
\[ L = \left[ \frac{3E}{2S} \right]^{1/2} \left[ D\Delta y \right]^{1/2} \]
GRAPHICAL DETERMINATION OF MINIMUM REQUIRED LENGTH OF OFFSET AS A FUNCTION OF ALLOWABLE STRAIN